PROPER AND IMPROPER MODAL SOLUTIONS INHOMOGENEOUS STRIPLINE

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ABSTRACT

A rigorous procedure is developed to determine the propagation constant for an *inhomogeneous* stripline, which consists of a perfectly conducting strip of infinitesimal thickness and finite width embedded in multiple dielectric layers between two perfectly conducting ground planes. An integral equation, formulated in terms of an electric field Green's function, is obtained by enforcing the boundary conditions on the strip. The current distribution and propagation constant are determined by solving the integral equation using a method of moments procedure. For several inhomogeneous stripline structures, both *proper* and *improper* dominant modal solutions are obtained. One of the most important practical cases, studied in detail, is that of the conventional stripline with an air-gap above the strip. This work represents the first reporting of improper modal solutions for such a stripline.

I. INTRODUCTION

Planar stripline transmission lines have been widely applied in microwave and millimeter-wave systems. The *inhomogeneous* stripline consists of multiple dielectric layers in which a thin perfectly conducting strip is embedded, bounded by conducting ground planes. Figure 1 illustrates the special case of a stripline with an air gap. The electrical properties of each layer are characterized by a complex permittivity and complex permeability. In design work, the transmission line parameters, propagation constant ($k_{xo} = \beta - j\alpha$) and characteristic impedance (Z_o), of the stripline are required. Previous investigators have characterized the dominant mode of many inhomogeneous striplines [1]. In these investigations, the propagation constant, for lossless structures, was always real and greater than the guided-mode wavenumbers of the background structure (i.e. the parallel-plate waveguide). These real k_{xo} wavenumbers correspond to *proper* modal solutions because the associated fields satisfy the radiation condition in the direction transverse to the strip. However, there may exist solutions for which the propagation constant is less than the guided-mode wavenumber of the dominant parallel-plate mode. Such a solution would correspond to a stripline mode which *leaks* into the parallel-plate waveguide mode. For these solutions, the propagation constant would be, in general, complex, corresponding to *improper* modal solutions. The fields associated with these improper solutions do not satisfy the radiation condition in the direction transverse to the strip. However, these improper solutions can be physically significant, for finite source distributions, in a restricted region about the strip. We have found both proper and improper modal solutions for the dominant modes of several inhomogeneous stripline configurations. We will present the results for the important problem of a stripline with a small air gap.

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II. FORMULATION

Our analysis of the inhomogeneous stripline is based on standard spectral domain techniques. The strip is assumed to be infinitesimally thin, located within the m'th layer, and oriented along the x direction. In order to simplify the analysis, we assume that the layered structure is infinite along the x and y coordinates so that the original three-dimensional analysis is reduced to a one-dimensional problem in the spectral domain by utilizing a two-dimensional Fourier transform in x and y. The width of the strip is assumed to be a small fraction of a wavelength; therefore, only longitudinal (x directed) currents are assumed, so that

$$\mathbf{J_s} = J_x(y)e^{-jk_{x_o}x}\hat{\mathbf{x}} \tag{1}$$

where $J_x(y)$ is the transverse variation of current and k_{x_o} is the complex modal propagation constant. An integral equation in terms of the Fourier transform of $J_x(y)$ is derived by enforcing the PEC boundary conditions on the strip,

$$\int_{-\infty}^{\infty} \tilde{G}_{xx}(k_{xo}, k_y, z_o)\tilde{J}_x^2(k_y)\, dk_y = 0, \tag{2}$$

where $\tilde{G}_{xx}$ is the spectral domain representation of the $\hat{\mathbf{x}}\hat{\mathbf{x}}$ component of the electric field dyadic Green's function. To formulate the electric field Green's function, Maxwell's equations are transformed, via a pair of Fourier transformations, into a pair of scalar transmission line equations.

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These transmission line expressions are solved by enforcing the boundary conditions at each interface separating the layers. This approach, often referred to as the *spectral domain immittance method*, is presented in detail in [2]. The integral equation (2) is solved using the method of moments. In this procedure the transverse current variation $J_x(y)$ is expanded in a series of basis functions; Maxwell-cosine and pulse functions are used in this investigation. The resulting homogeneous matrix equation is solved to obtain the complex modal propagation constant k_{xo} and the normalized current distribution. For a lossless structure k_{xo}^* also represents a valid solution to (2); however, for complex k_{xo} this corresponds to a nonphysical solution which increases exponentially in the direction of propagation.

III. CHOICE OF INTEGRATION PATH

In the proceeding analysis, the integration path in (2) has been left unspecified. The conventional choice is the real axis in the k_y-plane, extending from $-\infty$ to ∞. This path (or any path directly equivalent to it by Cauchy's theorem) is the correct one for obtaining solutions k_{xo} which correspond to *proper* modal solutions, in which the energy of the mode is confined in a region near the strip, and decays away from the strip in the y direction. This follows from the fact that proper modes are Fourier transformable functions, in the usual sense. Proper modal solutions are also, by necessity, *real* solutions, meaning that k_{xo} is real, for a lossless structure. A complex k_{xo} for these proper modes would correspond to attenuation as the wave propagates along the strip, which would violate the conservation of energy. Another property common to the proper solutions is that $k_{xo} \geq k_{TM_o}$. If k_{xo} were less than k_{TM_o}, corresponding poles in the k_y-plane would be located on the real axis. Such pole singularities would correspond to a nontransformable (and therefore unbounded) modal solution.

In addition to proper modal solutions, it is also interesting to explore the possibility of *improper* modal solutions (leaky-waves), in which case k_{xo} may be complex. It is well-known that complex leaky-wave solutions exist on a variety of guiding structures [3]. The complex nature of the wave solution generally corresponds to radiation into the medium surrounding the guiding structure. Although complex solutions must be improper, and thus violate the radiation condition at infinity, they may nevertheless be very important in explaining the field behavior in restrictive regions near the guiding structure, as discussed in [3].

For the case of an inhomogeneous stripline, such leakage, if it exists, must correspond to a launching of the dominant TM_o parallel-plate mode, since this is the only mechanism by which power can be propagated away from the strip (assuming all higher modes are below cutoff). The integral equation (2) may still be used to obtain the solution k_{xo} for the improper modes, provided the integration path is suitably chosen. For the stripline shown in Figure 1, the only propagating parallel-plate mode is assumed to be the TM_0 mode; therefore, the integration path is that shown in Figure 2 [4]. The deformed path is equivalent to the conventional real axis path, plus an additional part which encircles the k_{TM_o} poles resulting in a residue contribution. This residue contribution corresponds to the launching of the TM_o mode away from the strip.

IV. NUMERICAL RESULTS

In the practical realization of many striplines, it is very difficult to eliminate small air gaps in the structure. This often results in unexpected and frustrated transmission-line performance. The structure shown in Figure 1 is used to study the air-gap problem. As mentioned above, the width of the strip is assumed to be small compared to a wavelength. In addition, the dimensions of the stripline are such that only the TM_0 parallel-plate waveguide mode propagates. Thus, the transverse wavenumber for the TM_0 mode, k_{TM_0}, is purely real (assuming lossless materials), and the transverse wavenumbers for the other parallel-plate modes are purely imaginary, corresponding to modes below cutoff. These transverse wavenumbers (k_{tw}) are actually the poles of the Green's function $\tilde{G}_{xx}$. These poles are located at $k_{yp} = [k_{tw}^2 - k_{xo}^2]^{1/2}$ in the complex k_y-plane.

As an example, for the stripline shown in figure 1, the strip width w and the substrate thickness h are both taken as 0.1 cm, the substrate dielectric constant $\epsilon_r = 2.2$, and the operating frequency is 3 GHz. The resulting dominant mode propagation constants are shown in Figures 3 and 4, where Figure 3 is the plot of the real part of k_{xo} normalized with respect to k_0 (β/k_0) and Figure 4 is the plot of the normalized imaginary part of k_{xo} (α/k_0), versus the air gap thickness δ. In Figure 3, the values of β for both the proper and improper modal solutions are shown, along with a plot of k_{TM_0}. As seen in this figure, both β and k_{TM_0} decrease substantially with increasing δ. For a zero-thickness air gap ($\delta = 0$), the values of β for both the proper and improper solutions are equal to k_{TM_0}, which is the well-known solution for the homogeneous stripline. In this case, the poles corresponding to k_{TM_0} are removable singularities at the origin in the k_y-plane. Also apparent from Figure 3 is that β for the proper solution is always greater than or equal to k_{TM_0}; however, β for the improper solution is less than k_{TM_0} for small δ. For small values of δ, as shown in Figure 4, the attenuation constant α increases with air gap thickness. The attenuation constant reaches a maximum just before β crosses k_{TM_0}, and then decreases rapidly to zero. When α reaches zero the k_{xo} solution merges with the k_{xo}^* solution. At this point $k_{xo} = k_{xo}^*$, thus the solution must be real. Although real, this solution is still improper. For larger values of δ the improper solution splits into two real improper solutions.

To help distinguish between the proper and improper solutions it is desirable to examine the corresponding modal fields. For $\delta = 0.01$ cm, Figure 5 shows a plot of the electric field corresponding to the proper solution for the geometry of Figure 4. Figure 6 is a plot of the real part of the electric field for the improper solution, and Figure 7 is a plot of the imaginary part of the electric field for the improper solution. As seen from these figures, the field configurations are quite different. In fact, the field configuration for the *improper* solution corresponds to that associated with the homogeneous stripline, whereas that of the proper solution is quite different. This leads to the conclusion that, for striplines with small air gaps, it is the improper modal solution which corresponds to the conventional stripline mode.

V. CONCLUSIONS

A general and rigorous spectral domain formulation for the analysis of an arbitrary multiple-layer stripline has been developed. This formulation is used to determine the propagation constant, normalized surface current distribution, and corresponding fields for the dominant mode in the stripline structure. For multiple-layer striplines, both a proper and an improper dominant mode solution exist, in general. For the proper solution, the fields are confined to a region near the strip, and the propagation constant k_{xo}, for a lossless structure, is purely real and always greater than k_{TM_o}, the propagation constant of the lowest parallel-plate mode of the structure. The improper solution is generally a complex mode with $k_{xo} = \beta - j\alpha$, where β and α are greater than zero. This mode increases with distance from the strip, and represents a stripline mode which leaks into the dominant parallel-plate mode of the structure.

One specific structure which was examined in detail is that of a conventional homogeneous stripline in which a small air-gap is introduced above the strip. The inhomogeneity introduced by the air-gap results in the existence of both proper and improper dominant mode solutions, which have different propagation constants and field configurations. One of the most interesting aspects of the air-gap problem is the fact that the improper complex leaky-wave solution is the one which has fields that corresponds to those of a conventional homogeneous stripline. The newly identified improper modal solution may thus be significant in explaining abnormally high losses and unpredictable performance which are features of many practical striplines.

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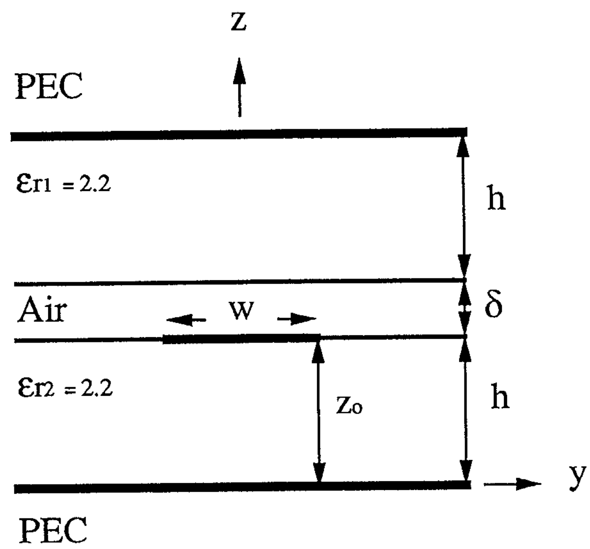

Figure 1: Stripline with an air-gap of thickness δ.

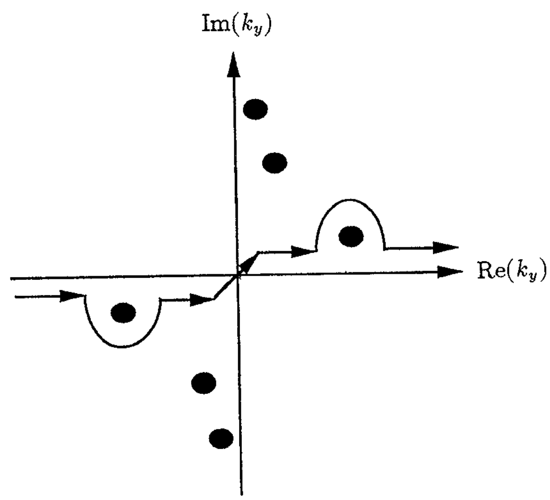

Figure 2: Integration path to obtain k_{xo} for the improper modal solution.

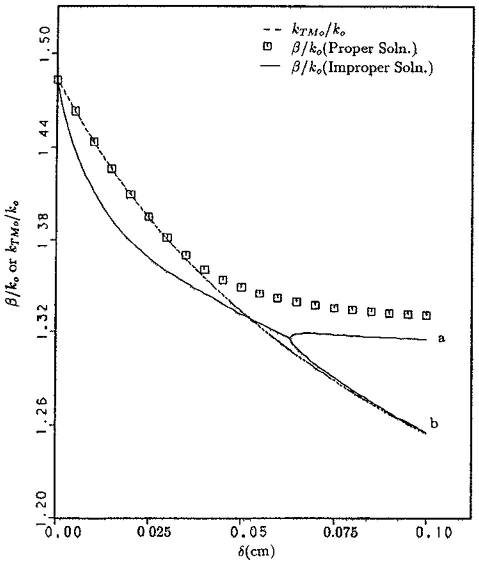

Figure 3: Normalized phase constant for proper and improper modal solutions, and k_{TM_0}/k_0 versus air-gap thickness, at 3 GHz ($h = w = 0.1$ cm), for the stripline of Fig. 1.

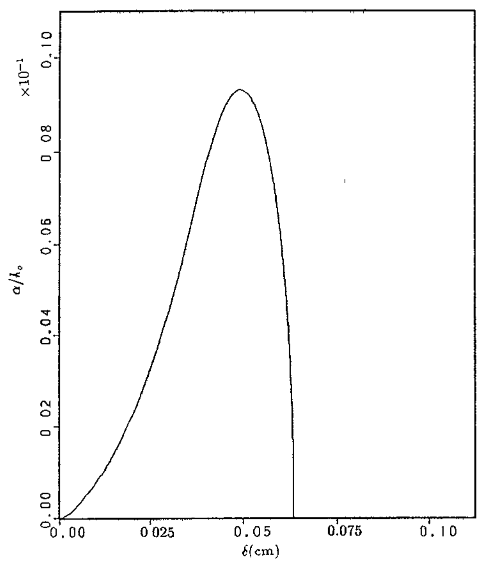

Figure 4: Normalized attenuation constant for the improper modal solution versus air-gap thickness, at 3 GHz ($h = w = 0.1$ cm), for the stripline of Fig. 1.

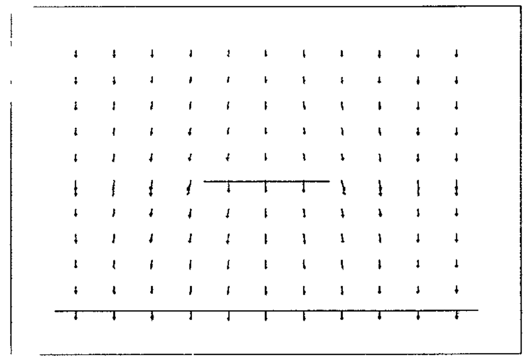

Figure 5: The real part of the electric field distribution for the proper solution ($\delta = 0.01$ cm), in a rectangular window of 0.2 cm $\times$ 0.3 cm around the center strip ($w = 0.1$ cm), for the geometry of Fig. 4.

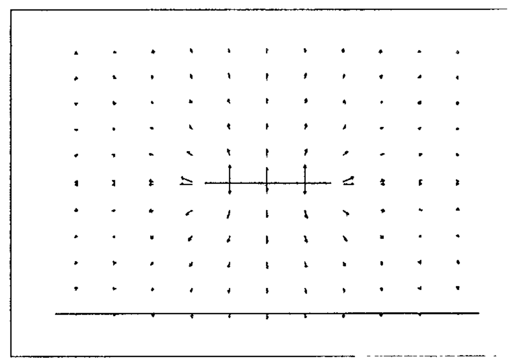

Figure 6: The real part of the electric field distribution for the improper solution ($\delta = 0.01$ cm), in a rectangular window of 0.2 cm $\times$ 0.3 cm around the center strip ($w = 0.1$ cm), for the geometry of Fig. 4.

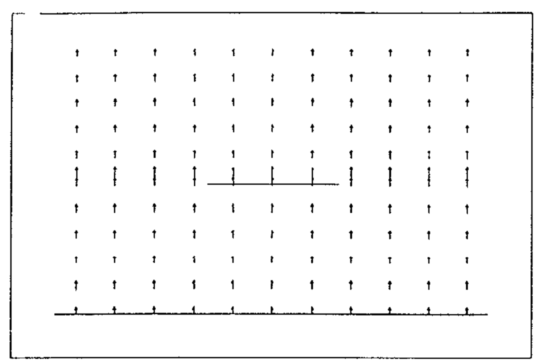

Figure 7: The imaginary part of the electric field distribution for the improper solution ($\delta = 0.01$ cm), in a rectangular window of 0.2 cm $\times$ 0.3 cm around the center strip ($w = 0.1$ cm), for the geometry of Fig. 4.